# Supplementary Material for: “Reflective optical vortex generators with ultrabroadband self-phase compensation”

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## 1 The derivation of the Jones matrix of single-twisted self-phase compensated device

For a twisted liquid crystal (LC) layer with thickness $d$ and twist angle $\phi$, the Jones Matrix in the local principal coordinate (x component is along the LC director and y component is perpendicular to the director) is given by

$$\mathbf{M} = \begin{pmatrix} \cos X - i\frac{\Gamma}{2}\frac{\sin X}{X} & \phi\frac{\sin X}{X} \\ -\phi\frac{\sin X}{X} & \cos X + i\frac{\Gamma}{2}\frac{\sin X}{X} \end{pmatrix}, \tag{S1}$$

where $X=\sqrt{(\Gamma/2)^2+\phi^2}$ and $\Gamma = 2\pi\Delta nd/\lambda$ is retardation with $\Delta n$, and $\lambda$ being the birefringence, and incident wavelength. A single-twist self-phase compensated device is regarded as a reflective waveplate with space-varying optical axis. In the fixed coordinate, the corresponding Jones matrix is thus given by

$$\mathbf{T}_1 = \mathbf{R}(-\alpha)\mathbf{M}(d,-\phi)\mathbf{M}(d,\phi)\mathbf{R}(\alpha), \tag{S2}$$

where $\mathbf{R}(\alpha)$ is rotation matrix with rotation angle $\alpha$. To reduce the complexity, we first consider the combined matrix

$$\mathbf{M}(d,-\phi)\mathbf{M}(d,\phi)=\begin{pmatrix}\cos X-i\frac{\Gamma}{2}\frac{\sin X}{X} & -\phi\frac{\sin X}{X}\\ \phi\frac{\sin X}{X} & \cos X+i\frac{\Gamma}{2}\frac{\sin X}{X}\end{pmatrix}\begin{pmatrix}\cos X-i\frac{\Gamma}{2}\frac{\sin X}{X} & \phi\frac{\sin X}{X}\\ -\phi\frac{\sin X}{X} & \cos X+i\frac{\Gamma}{2}\frac{\sin X}{X}\end{pmatrix}$$
$$=\begin{pmatrix}\cos^2 X+\left[\phi^2-\left(\frac{\Gamma}{2}\right)^2\right]\frac{\sin^2 X}{X^2}-i\Gamma\frac{\sin X\cos X}{X} & -i\Gamma\phi\frac{\sin^2 X}{X^2}\\ -i\Gamma\phi\frac{\sin^2 X}{X^2} & \cos^2 X+\left[\phi^2-\left(\frac{\Gamma}{2}\right)^2\right]\frac{\sin^2 X}{X^2}+i\Gamma\frac{\sin X\cos X}{X}\end{pmatrix}, \tag{S3}$$

Define

$$A_1=\cos^2 X+\left[\phi^2-\left(\frac{\Gamma}{2}\right)^2\right]\frac{\sin^2 X}{X^2}, \tag{S4}$$

$$A_2=\Gamma\frac{\sin X\cos X}{X}, \tag{S5}$$

$$A_3=-\Gamma\phi\frac{\sin^2 X}{X^2}. \tag{S6}$$

We get

$$\mathbf{M}(d,-\phi)\mathbf{M}(d,\phi)=\begin{pmatrix}A_1-iA_2 & iA_3\\ iA_3 & A_1+iA_2\end{pmatrix}. \tag{S7}$$

Substituting Equation (S7) into Equation (S2), we have

$$\mathbf{T}_1=A_1\begin{pmatrix}1&0\\0&1\end{pmatrix}-i\begin{pmatrix}A_2\cos 2\alpha+A_3\sin 2\alpha & A_2\sin 2\alpha-A_3\cos 2\alpha\\ A_2\sin 2\alpha-A_3\cos 2\alpha & -(A_2\cos 2\alpha+A_3\sin 2\alpha)\end{pmatrix}. \tag{S8}$$

Defining

$$e^{i\varphi_1}=\frac{1}{\sqrt{A_2^{\,2}+A_3^{\,2}}}\left(A_2+iA_3\right), \tag{S9}$$

we get the final expression as follows

$$\mathbf{T}_1=A_1\begin{pmatrix}1&0\\0&1\end{pmatrix}-i\sqrt{A_2^{\,2}+A_3^{\,2}}\begin{pmatrix}\cos(2\alpha-\varphi_1) & \sin(2\alpha-\varphi_1)\\ \sin(2\alpha-\varphi_1) & -\cos(2\alpha-\varphi_1)\end{pmatrix}$$
$$=\left\{\cos^2 X+\left[\phi^2-\left(\frac{\mathit{\Gamma}}{2}\right)^2\right]\frac{\sin^2 X}{X^2}\right\}\begin{pmatrix}1&0\\0&1\end{pmatrix}-i\mathit{\Gamma}\frac{\sin X}{X}\sqrt{\cos^2 X+\phi^2\frac{\sin^2 X}{X^2}}\begin{pmatrix}\cos(2\alpha-\varphi_1) & \sin(2\alpha-\varphi_1)\\ \sin(2\alpha-\varphi_1) & -\cos(2\alpha-\varphi_1)\end{pmatrix}. \tag{S10}$$

## 2 The derivation of the Jones matrix of dual-twisted self-phase compensated device

The dual-twisted self-phase compensated device has two twisted LC layers with continuously variable optical axis. Therefore, we have

$$\mathbf{T}_2=\mathbf{R}(-\alpha)\mathbf{M}(d_1,-\phi_1)\mathbf{M}(d_2,-\phi_2)\mathbf{M}(d_2,\phi_2)\mathbf{M}(d_1,\phi_1)\mathbf{R}(\alpha), \tag{S11}$$

where $d_\mathrm{n}$ and $\phi_\mathrm{n}$ are the thickness and twist angle of n^th LC layer, respectively. The Equation (S11) has tedious expression, so we first consider

$$\mathbf{M}_{\text{twisted}}(d_2,\phi_2)\mathbf{M}_{\text{twisted}}(d_1,\phi_1)=\begin{pmatrix} B_1-iB_2 & B_3+iB_4 \\ -B_3+iB_4 & B_1+iB_2 \end{pmatrix}, \tag{S12}$$

where

$$B_1=\cos X_1\cos X_2-\left(\frac{\Gamma_1\Gamma_2}{4}+\phi_1\phi_2\right)\frac{\sin X_1\sin X_2}{X_1X_2}, \tag{S13}$$

$$B_2=\cos X_1\frac{\Gamma_2}{2}\frac{\sin X_2}{X_2}+\cos X_2\frac{\Gamma_1}{2}\frac{\sin X_1}{X_1}, \tag{S14}$$

$$B_3=\cos X_1\phi_2\frac{\sin X_2}{X_2}+\cos X_2\phi_1\frac{\sin X_1}{X_1}, \tag{S15}$$

$$B_4=\left(\frac{\Gamma_1}{2}\phi_2-\frac{\Gamma_2}{2}\phi_1\right)\frac{\sin X_1\sin X_2}{X_1X_2}, \tag{S16}$$

where subscript 1, 2 denote the n^th LC layer. Notice that $\mathbf{M}(d_1,-\phi_1)\mathbf{M}(d_2,-\phi_2)$ can be obtained by interchanging subscripts of $d$ and $\phi$ from Equation (S16), so we have the following

$$\mathbf{M}(d_1,-\phi_1)\mathbf{M}(d_2,-\phi_2)=\begin{pmatrix} B_1-iB_2 & -B_3+iB_4 \\ B_3+iB_4 & B_1+iB_2 \end{pmatrix}. \tag{S17}$$

And we get

$$\begin{aligned}&\mathbf{M}(d_1,-\phi_1)\mathbf{M}(d_2,-\phi_2)\mathbf{M}(d_2,\phi_2)\mathbf{M}(d_1,\phi_1)\\&=\begin{pmatrix} B_1^2-B_2^2+B_3^2-B_4^2-i2(B_1B_2+B_3B_4) & i2(B_1B_4-B_2B_3) \\ i2(B_1B_4-B_2B_3) & B_1^2-B_2^2+B_3^2-B_4^2+i2(B_1B_2+B_3B_4) \end{pmatrix}\end{aligned} \tag{S18}$$

Define

$$C_1=B_1^2-B_2^2+B_3^2-B_4^2, \tag{S19}$$

$$C_2=2(B_1B_2+B_3B_4), \tag{S20}$$

$$C_3=2(B_1B_4-B_2B_3). \tag{S21}$$

We get

$$\mathbf{M}(d_1,-\phi_1)\mathbf{M}(d_2,-\phi_2)\mathbf{M}(d_2,\phi_2)\mathbf{M}(d_1,\phi_1)=\begin{pmatrix} C_1-iC_2 & iC_3 \\ iC_3 & C_1+iC_2 \end{pmatrix}, \tag{S22}$$

Notice that Equation (S22) has the same mathematical form as Equation (S7), we can directly obtain

$$\mathbf{T}_2=C_1\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}-i\sqrt{C_2^{\,2}+C_3^{\,2}}\begin{pmatrix} \cos(2\alpha-\varphi_2) & \sin(2\alpha-\varphi_2) \\ \sin(2\alpha-\varphi_2) & -\cos(2\alpha-\varphi_2) \end{pmatrix}, \tag{S23}$$

where

$$e^{i\varphi_2} = \frac{1}{\sqrt{C_2{}^2 + C_3{}^2}}\left(C_2 + iC_3\right) \tag{S24}$$

The conversion efficiency is

$$\eta_2 = 4(B_1^2 + B_3^2)(B_2^2 + B_4^2) \tag{S25}$$

The dual-twisted self-compensated FPG we demonstrate is designed in the band of 430 – 900 nm. The calculated thicknesses and twist angles are $d_1 = 2.25$ μm, $d_2 = 1.06$ μm, $\phi_1 = 102.5°$ and $\phi_2 = -66.9°$.